$$I(K) = \sum_{i=1}^{n} \lceil c_i k \rceil$$

$$M = \left[\lfloor \beta^{-1}(I_0) \rfloor , \lceil \alpha^{-1}(I_0) \rceil \right]$$

$$\alpha^{-1}(k) = \frac{K}{\sum_{i=1}^{n} c_i} \qquad \beta^{-1}(k) = \frac{k-n}{\sum_{i=1}^{n} c_i}$$

original Example:

$$I_0 = 192 \qquad C = \left\{ \frac{7}{100}, \frac{22}{100}, \frac{2}{200}, \frac{1}{100}, \frac{10}{50}, \frac{10}{100} \right\}$$

$$= \left\{ \frac{7}{100}, \frac{22}{100}, \frac{1}{100}, \frac{1}{100}, \frac{20}{100}, \frac{10}{100} \right\}$$

$$\sum_{i=1}^{n} c_i = (7 + 22 + 1 + 1 + 20 + 10)/100 = {}^{61}\!/_{100}$$

$$\alpha^{-1}(192) = {}^{192}\!/_{(61/100)} = \frac{192 \times 100}{61} = \frac{19,200}{61}$$

$$\lceil \alpha^{-1}(192) \rceil = 315$$

$$\beta^{-1}(192) = \frac{(192-6) \times 100}{61} = \frac{18,600}{61}$$

$$\lfloor \beta^{-1}(192) \rfloor = 304 \qquad M = [304, 315]$$

$$I(304) = \left\lceil \frac{7 \times 304}{100} \right\rceil + \left\lceil \frac{22 \times 304}{100} \right\rceil + \left\lceil \frac{1 \times 304}{100} \right\rceil$$
$$+ \left\lceil \frac{1 \times 304}{100} \right\rceil + \left\lceil \frac{20 \times 304}{100} \right\rceil + \left\lceil \frac{10 \times 304}{100} \right\rceil$$
$$= 22 + 67 + 4 + 4 + 61 + 31 = 189$$

$$I(315) = \left\lceil \frac{7 \times 315}{100} \right\rceil + \left\lceil \frac{22 \times 315}{100} \right\rceil + \left\lceil \frac{1 \times 315}{100} \right\rceil$$
$$+ \left\lceil \frac{1 \times 315}{100} \right\rceil + \left\lceil \frac{20 \times 315}{100} \right\rceil + \left\lceil \frac{10 \times 315}{100} \right\rceil$$
$$= 23 + 70 + 4 + 4 + 63 + 32 = 196$$

$$I(310) = 22 + 69 + 4 + 4 + 62 + 31 = 192$$

$$I(k) = \sum_{i=1}^{n} \lceil c_i k \rceil$$

$$M = \left[\lfloor \beta^{-1}(I_0) \rfloor, \lceil \alpha^{-1}(I_0) \rceil \right]$$

$$\alpha^{-1}(k) = \frac{k}{\sum_{i=1}^{n} c_i} \qquad \beta^{-1}(k) = \frac{k - n}{\sum_{i=1}^{n} c_i}$$

Minecraft Cake example:

$$I_0 = 36 \qquad \overset{\text{wheat}\quad\text{sugar}\quad\text{egg}\quad\text{milk}}{C = \left\{ \frac{3}{64}, \frac{2}{64}, \frac{1}{16}, \frac{3}{1} \right\}}$$

$$\underset{=4/64}{} \qquad \underset{=192/64}{}$$

$$\sum_{i=1}^{n} c_i = (3 + 2 + 4 + 192)/64 = 201/64$$

$$\alpha^{-1}(36) = \frac{36 \times 64}{201} = \frac{2304}{201}$$

$$\lceil \alpha^{-1}(36) \rceil = 12$$

$$\beta^{-1}(36) = \frac{(36 - 4) \times 64}{201} = \frac{2048}{201}$$

$$\lfloor \beta^{-1}(36) \rfloor = 10$$

$$I(10) = \lceil 30/64 \rceil + \lceil 20/64 \rceil + \lceil 10/16 \rceil + 30 = 33$$

$$I(12) = \lceil 36/64 \rceil + \lceil 24/64 \rceil + \lceil 12/16 \rceil + 36 = 39$$

$$I(11) = \lceil 33/64 \rceil + \lceil \tfrac{22}{64} \rceil + \lceil 11/16 \rceil + 33 = 36$$