Maximizing Batch Crafting - Icy Mink 16 Feb 2021 (Corrections done 5 January 2025)

Suppose that there is an item you want to batch craft, B, that is crafted using n unique items, and you have I_0 inventory slots. Let $d_1, d_2, \ldots, d_n$ be the amount of a certain item used in the crafting where d_i is the ith item in the list. Let $s_1, s_2, \ldots, s_n$ be how many of a certain item can be in an item stack. Let $c_1, c_2, \ldots, c_n$ be the constants derived as such:

$$c_i = \frac{d_i}{s_i}$$

We will define a function $I(k)$ that returns the amount of inventory slots used if you want to craft the item B, k times:

$$I(k) = \sum_{i=1}^{n} \lceil c_i k \rceil$$

The ceiling function is applied since having a single item requires an inventory slot. Now let us define the function $\alpha(k)$ as an approximation of I:

$$\alpha(k) = k \cdot \sum_{i=1}^{n} c_i$$

The properties of the ceiling function, namely that $f(k) + 1 \geq \lceil f(k) \rceil \geq f(k)$, we can apply this fact to get the other approximation function $\beta(k)$:

$$\beta(k) = \alpha(k) + n$$

$$\beta(k) \geq I(k) \geq \alpha(k)$$

So we know that the solution will be somewhere in this range. The inverse of the approximation functions are as follows:

$$\alpha^{-1}(k) = \frac{k}{\sum_{i=1}^{n} c_i} \quad \beta^{-1}(k) = \frac{k-n}{\sum_{i=1}^{n} c_i}$$

$$\alpha^{-1}(k) \geq I^{-1}(k) \geq \beta^{-1}(k)$$

We can now define an interval M:

$$M = [\lfloor \beta^{-1}(I_0) \rfloor, \lceil \alpha^{-1}(I_0) \rceil]$$

The largest value of k such that $I(k) \leq I_0$, which would be $I^{-1}(I_0)$, is within the interval M.

[ANECDOTE] Another approximation function uses the Fourier Series associated with the function. The Fourier Series of $\lceil x \rceil$ is:

$$\lceil x \rceil \approx x + \frac{1}{2} + \frac{1}{\pi} \sum_{i=1}^{\infty} \frac{\sin(2\pi i x)}{i}$$

If we generalize this for $\lceil cx \rceil$ for any constant c we get:

$$\lceil cx \rceil \approx cx + \frac{1}{2} + \frac{c}{\pi} \sum_{i=1}^{\infty} \frac{\sin(2\pi c i x)}{ci}$$

And thus an alternative approximation for $I(k)$ is:

$$I(k) \approx f(k) = \sum_{i=1}^{n} \left(c_i k + \frac{1}{2} + \frac{c_i}{\pi} \sum_{j=1}^{\infty} \frac{\sin(2\pi c_i i k)}{c_i i} \right)$$

While this approximation is highly accurate, the infinite sum diverges so the upper bound would need to be lowered, and finding a sufficient upper bound that is high enough for accuracy's sake is difficult, and the upper bound must be higher for a higher n. In conclusion, while this approximation is much more accurate, the amount of computation is so high that it is not practical. Even if a program were written to compute this, it would be absurdly slow, thus we will not use this approximation for solving this problem.