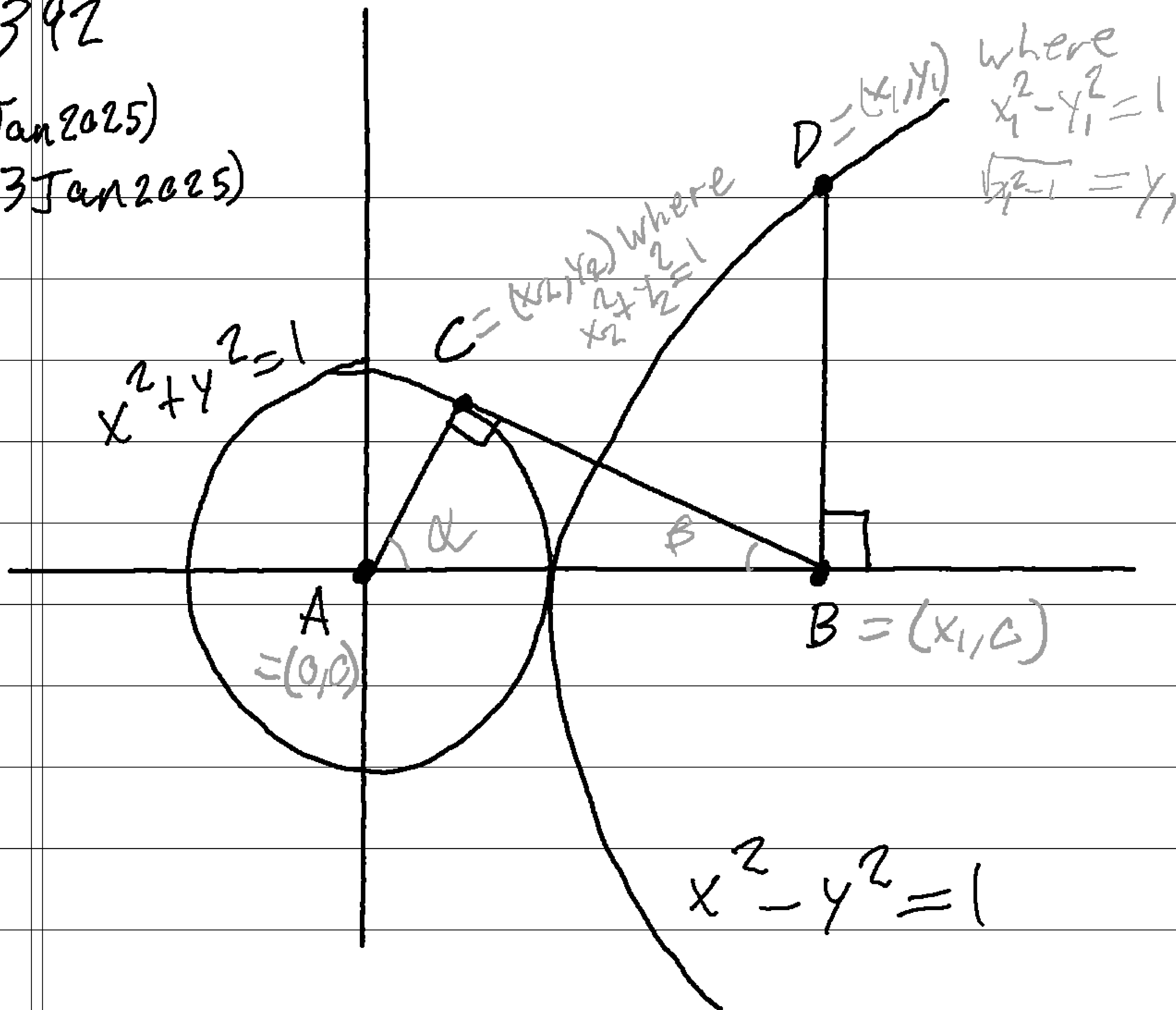
PME #1392
(Started 2 Jan 2025)
(Finished 3 Jan 2025)



Show $\|\overline{DB}\| = \|\overline{CB}\|$:

$D=(x_1,y_1)$ where $x_1^2-y_1^2=1 \Rightarrow y_1=\sqrt{x_1^2-1}=\|\overline{DB}\|$

$B=(x_1,0)$, $C=(x_2,y_2)$ where $x_2^2+y_2^2=1$

$\|\overline{CB}\|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(x_2-x_1)^2+y_2^2}$

Since the radius of the circle is 1, $\|\overline{AC}\|=1$

Since $B=(x_1,0)$ and $A=(0,0)$, $\|\overline{AB}\|=x_1$.

Thus by the Pythagorean Theorem:

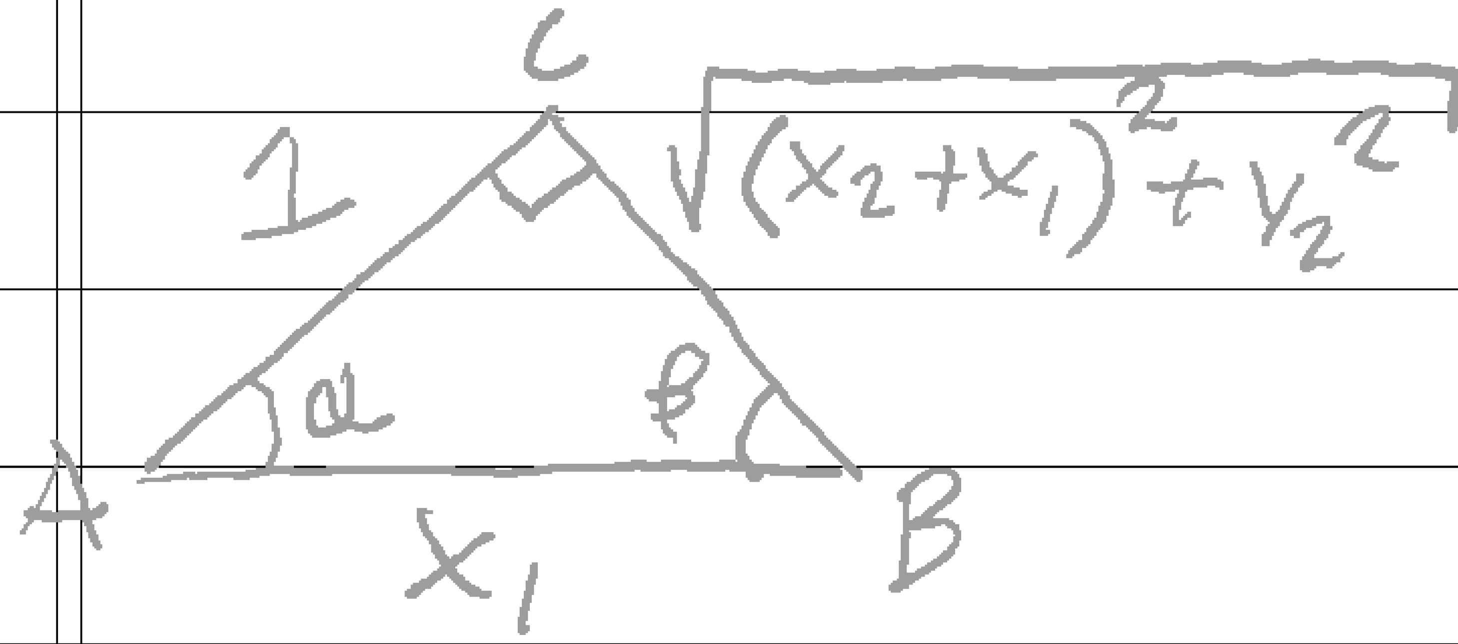
$$1^2+\left(\sqrt{(x_2-x_1)^2+y_2^2}\right)^2=x_1^2$$

$$\Rightarrow \left(\sqrt{(x_2-x_1)^2+y_2^2}\right)^2=x_1^2-1$$

$$\Rightarrow \sqrt{(x_2-x_1)^2+y_2^2}=\sqrt{x_1^2-1}$$

$$\Rightarrow \|\overline{CB}\|=\|\overline{DB}\| \qquad \blacksquare$$

Show $\sqrt{(x_2+x_1)^2+y_2^2} = \sqrt{x_1^2-1}$



$$\left(\sqrt{(x_2+x_1)^2+y_2^2}\right)^2 + 1^2 = x_1^2$$

$$\Rightarrow \sqrt{(x_2+x_1)^2+y_2^2}^{\,2} + 1 = x_1^2$$

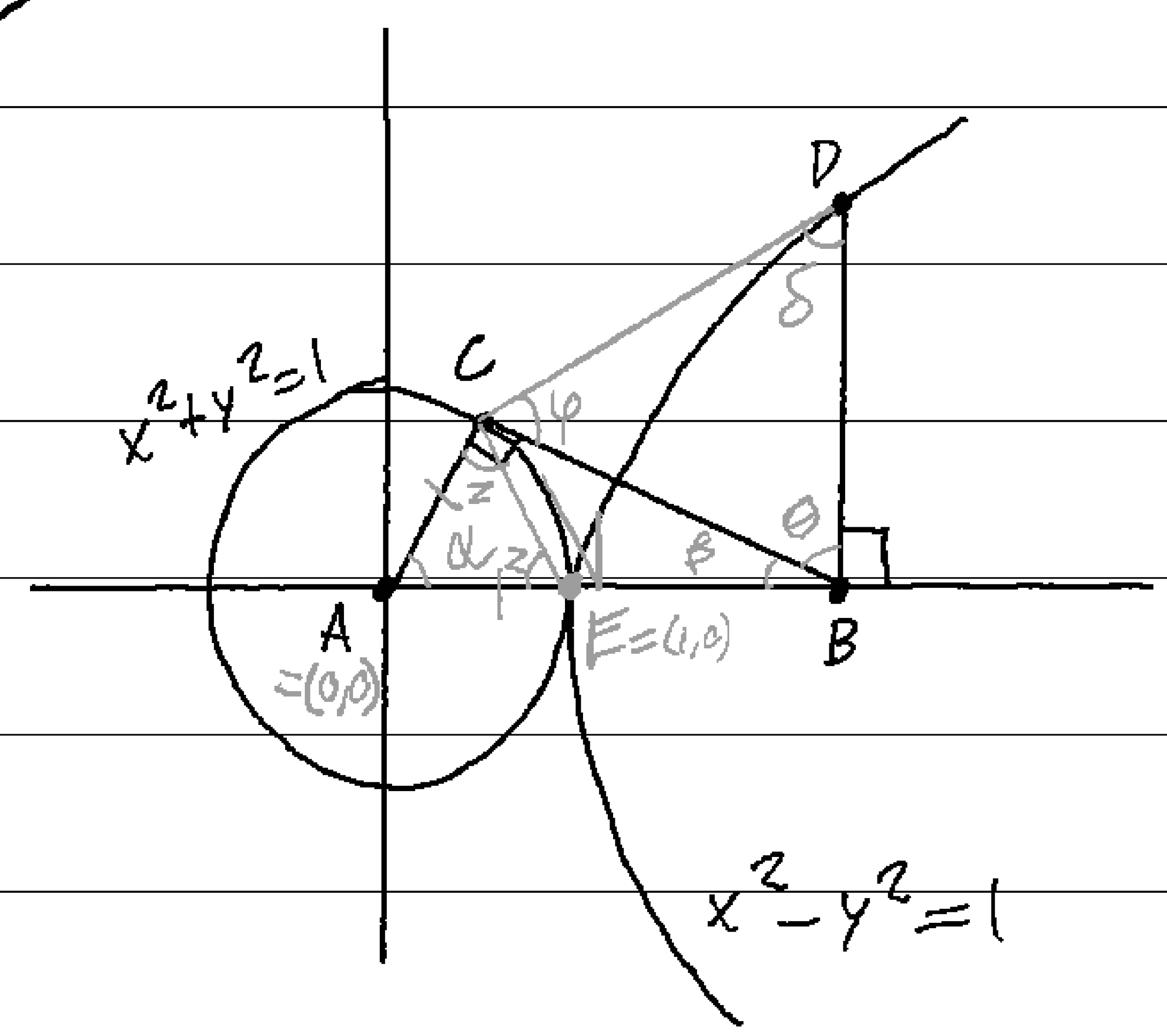
$$\Rightarrow \sqrt{(x_2+x_1)^2+y_2}^{\,2} = x_1^2 - 1$$

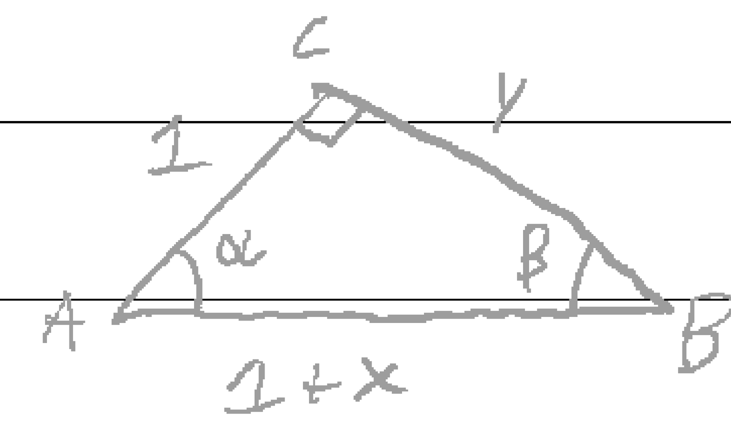
$$\Rightarrow \sqrt{(x_2+x_1)^2+y_2} = \sqrt{x_1^2-1}$$

~~If $\varphi = \delta = \theta$ then $\triangle BCD$ is equilateral~~

~~If $z = 60^\circ \Rightarrow \alpha = 60^\circ \Rightarrow \beta = 30^\circ \Rightarrow \theta = 60^\circ$~~

$\Rightarrow$ for small x in D, $\triangle BCD$ is not equilateral





$y^2+1^2=(1+x)^2$

$y^2+1 = 1+2x+x^2$

$y^2 = x(2+x)$

$\sin(\alpha) = \frac{y}{1+x}$ $\quad$ $\sin(\beta) = \frac{1}{1+x}$

$\cos(\alpha) = \frac{1}{1+x}$ $\quad$ $\cos(\beta) = \frac{y}{1+x}$

$\tan(\alpha) = y$ $\quad$ $\tan(\beta) = 1/y$

$2x+\alpha = 180$

$\Rightarrow x = \frac{180-\alpha}{2}$

$x = 90 - \frac{\alpha}{2}$

$\alpha < 90 \Rightarrow \frac{\alpha}{2} < 45$

$\Rightarrow 45 + \frac{\alpha}{2} < 90$

$\Rightarrow 45 < 90 - \frac{\alpha}{2} = x$

$\Rightarrow x > 45^\circ$

$x^2 - y^2 = x^2 + y^2$

$-x^2 \quad\quad -x^2$

$\Rightarrow -(y^2) = y^2$

$\Rightarrow 0 = 2y^2$

$\Rightarrow 0 = y^2$

$\Rightarrow 0 = y$

$z + E = 180$

$z + c' = 90 \Rightarrow c' - z = 90 - 2z$

$E + c' + \beta = 180$

$z + E = E + c' + \beta$

$z = c' + \beta$

$0 = c' - z + \beta$

$0 = 90 - 2z + \beta$

$2z = 90 + \beta$

$z = 45 + \frac{\beta}{2}$ $\quad$ $\beta < 90^\circ$

$\frac{\beta}{2} < 45^\circ$

$z = 45 + \frac{\beta}{2} < 90^\circ$

$\alpha + \beta = 90^\circ$ and $\beta + \theta = 90^\circ \Rightarrow \alpha = \theta \Rightarrow \alpha < 90^\circ, \theta < 90^\circ$

$E = (x,y)$ where $x^2+y^2 = x^2-y^2 \Rightarrow y = 0 \Rightarrow x = 1 \Rightarrow E = (1,0)$

$\Rightarrow \|\overline{AC}\| = 1 = \|\overline{AE}\| \Rightarrow \triangle ACE$ is isosceles

$\Rightarrow \angle CEA = \angle ACE$ (let $\angle CEA = x$)

$\Rightarrow 2z + \alpha = 180^\circ \Rightarrow z = 90 - \frac{\alpha}{2}$ $(\alpha < 90^\circ) \Rightarrow z > 45^\circ$

Since z is a part of $\angle ACB$, $z < 90^\circ \Rightarrow 45^\circ < z < 90^\circ$